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SONIC VELOCITY IN TWO-PHASE SYSTEMS†

D. L. NGUYEN, E. R. F. WINTER and M. GREINER
Lehrstuhl C für Thermodynamik der Technischen Universität München, Arcisstr. 21, D-8000 München 2, B.R.D.

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Abstract—Simple relations for the prediction of the propagation of pressure disturbances in liquid–gaseous two-phase systems are presented. The model applied makes use of the well known physical behaviour that the sonic velocity in a single-phase fluid is influenced by the elasticy of the confining walls. The novel concept is to consider the interface of the one phase to act as the elastic wall of the other phase and vice versa. The predictions comply well with the experimental data found in the literature.

INTRODUCTION

Sonic velocity as a material property of single-phase fluids has been studied for a long time and as such been applied to two-phase flows resulting in a multitude of computational models for the determination of the propagation of sound. However, to the knowledge of the authors no model exists allowing the prediction of the sonic velocity in two-phase flows over the entire range of the void fraction and for all flow regimes.

A single phase fluid flowing in a tube with an elastic wall shows a dependency upon the bulk modulus of the tube wall, i.e. the sonic velocity decreases with an increasing elasticity of the wall material. This fact is applied to two-phase systems, treating the interface of one phase as the elastic boundary of the other. It is assumed that no phase change occurs during the propagation of sound. The two-phase flow system is considered to be confined by a rigid wall.

ANALYSIS

The analysis of the propagation of an infinitesimal pressure disturbance in a pure fluid leads to the well known Laplace-equation for the sonic velocity a

$$a^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho} \qquad [1]$$

where p is the pressure and ρ the density. The propagation of an infinitesimal pressure wave in a pure fluid confined by an elastic tube wall was first described by Löwy (1928) and again by Raabe (1960) by the following equation

$$a_E{}^2 = \frac{a^2}{1 + \frac{E_F}{E_W}\frac{D}{S}}. \qquad [2]$$

Figure 1 depicts the effective sonic velocity, a_E, of a single phase pure fluid, bounded by an elastic wall, as a function of the bulk modulus E_W of the wall material. E_F is the bulk modulus of the fluid, D is the tube diameter and S the tube wall thickness. The theory for two-phase systems is based upon the following assumptions:

—The interface of the one phase acts as the elastic wall of the other and vice versa.
—No phase change occurs during the propagation of a pressure disturbance.
—Frictional forces are neglegible.
—No influence of the surface tension upon the pressure disturbance exists.
—The system is one-dimensional.
—The two-phase fluid is bounded by a rigid wall.

†Extract from a forthcoming doctoral thesis by Dipl.-Ing. D. L. Nguyen.

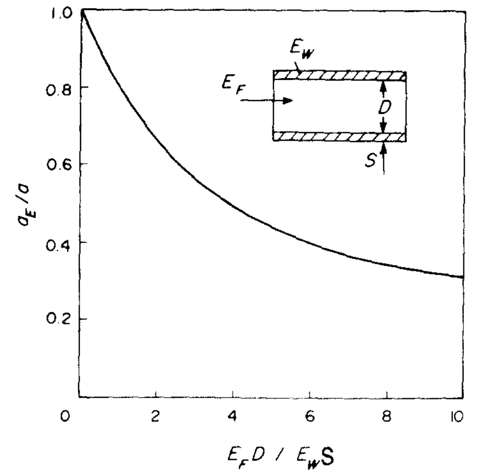

Figure 1. Sonic velocity in a pure fluid as function of the bulk modulus of the tube wall.

First the sonic velocities of the participating phases are derived employing the conservation equations for mass and momentum. Considering a stationary wave front in a moving single phase medium (figure 2) the continuity equation can be written as

$$\frac{\mathrm{d}w}{w}+\frac{\mathrm{d}F}{F}+\frac{\mathrm{d}\rho}{\rho}=0 \tag{3}$$

where w is the velocity and F is the cross section. Note, the term $\mathrm{d}F/F$ is customarily omitted, however in the current application the influence of the variation of the flow cross-section upon the propagation of the pressure disturbance is taken into account.

Neglecting the frictional forces the momentum equation can be written as

$$\rho w\,\mathrm{d}w+\mathrm{d}p=0\,. \tag{4}$$

Combination of the continuity and momentum equations yields

$$w^2=\frac{1}{\dfrac{\rho}{F}\dfrac{\mathrm{d}F}{\mathrm{d}p}+\dfrac{\mathrm{d}\rho}{\mathrm{d}p}}=a_E^2\,. \tag{5}$$

The above expression describes the effective sonic velocity of the individual phases confined by elastic boundaries. This effective sonic velocity depends upon the cross-sectional variation of

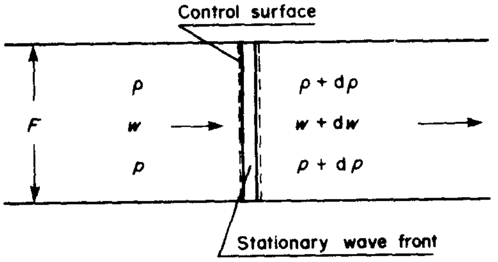

Figure 2. Propagation of an infinitesimal pressure pulse.

the flow caused by a pressure change and upon the sonic velocity as a physical property of the fluid under investigation.

Subsequently the effective sonic velocities will be developed for three different two-phase flow regimes.

Stratified flow model

Note, that the sonic velocities (physical properties) of the pure phases, a_L and a_G, are different from the effective sonic velocities, $a_{E,L}$ and $a_{E,G}$ in the two phase system.

Equation [5] is written for the gaseous phase in a stratified system (figure 3):

$$a_{E,G}^2 = \frac{1}{\frac{\rho_G}{F_G}\frac{dF_G}{dp} + \frac{d\rho_G}{dp}}. \qquad [6]$$

For a composite system bounded by a rigid tube the variation of the cross-sectional areas with pressure change is

$$\begin{aligned} dF &= dF_L + dF_G = 0 \\ dF_G &= -dF_L. \end{aligned} \qquad [7]$$

In stratified systems the differential of the cross-sectional area fraction equals the differential of the volume fraction:

$$d\left(\frac{F_L}{F}\right) = \frac{dF_L}{F} = \frac{dV_L}{V}$$

where V is the volume. Under the assumption that no phase change occurs, this relation results in

$$\frac{dF_L}{F} = \frac{M_L}{V} dv_L \qquad [8]$$

where v is the specific volume and M is the mass. Substitution of the specific volume into [1] leads to

$$dv_L = -\frac{v_L^2}{a_L^2} dp. \qquad [9]$$

Introducing [9] in [8] yields

$$\frac{dF_L}{F} = -\frac{M_L v_L^2}{V a_L^2} dp = \frac{-V_L}{V \rho_L a_L^2} dp. \qquad [10]$$

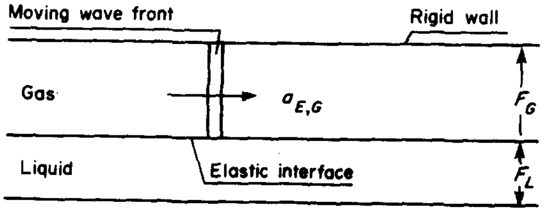

Figure 3. Propagation of an infinitesimal pressure pulse in the gas phase of a stratified two-phase system.

Combination of [10] and [7] gives

$$\frac{\mathrm{d}F_G}{\mathrm{d}p}=\frac{F_L}{\rho_L a_L^2}. \qquad [11]$$

Substitution of [11] and [1] into [6] leads to

$$a_{E,G}^2=\frac{1}{\frac{1}{a_G^2}+\frac{\rho_G}{\rho_L}\frac{F_L}{F_G}\frac{1}{a_L^2}}$$

which in turn by replacing of the area ratio by the void fraction $\alpha = F_G/F$ results in

$$a_{E,G}^2=\frac{1}{\frac{1}{a_G^2}+\frac{1-\alpha}{\alpha}\frac{\rho_G}{\rho_L}\frac{1}{a_L^2}}. \qquad [12]$$

The influence of the liquid phase upon the magnitude of the effective sonic velocity in the gaseous phase in the above equation is given by the term $\rho_L \cdot a_L^2$, which represents the compressibility of the liquid. Since the product of density and sonic velocity of the liquid is very large the second term in the denominator in [12] contributes very little.

The considerations which lead to the derivations of the effective sonic velocity in the gas phase are now applied to the liquid phase in an analogous way (figure 4). Equation [5] is written for the liquid phase:

$$a_{E,L}^2=\frac{1}{\frac{\rho_L}{F_L}\frac{\mathrm{d}F_L}{\mathrm{d}\rho}+\frac{\mathrm{d}\rho_L}{\mathrm{d}p}}. \qquad [13]$$

The corresponding expressions from [7] to [10] lead to

$$\frac{\mathrm{d}F_L}{\mathrm{d}p}=\frac{F_G}{\rho_G a_G^2}. \qquad [14]$$

Substitution of [1] and [14] into [13] yields the final relation for the effective sonic velocity in the liquid phase

$$a_{E,L}^2=\frac{1}{\frac{1}{a_L^2}+\frac{\alpha}{1-\alpha}\frac{\rho_L}{\rho_G}\frac{1}{a_G^2}}. \qquad [15]$$

In contrast to [12] the compressibility of the gas, $\rho_G \cdot a_G^2$, in the second term of the denomina-

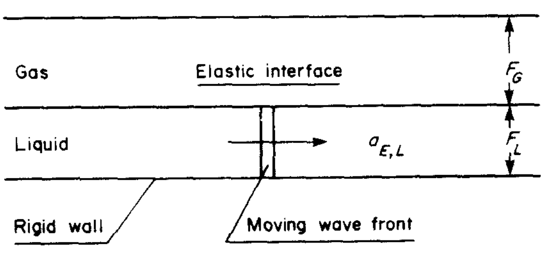

Figure 4. Propagation of an infinitesimal pressure pulse in the liquid phase of a stratified two phase system.

tor is quite small and thus exercises a larger influence upon the effective sonic velocity in the liquid phase.

For stratified one dimensional flow a composite sonic velocity does not exist, because each of the separated phases is continuous in axial direction. If a pressure pulse is imposed on the liquid and the gas at the same time, the disturbance propagates with different velocities in both phases in axial direction (parallel to the interface).

Slug flow model

Following Henry *et al.* (1971) the real distribution of the two phases in the slug flow pattern, as illustrated in figure 5(a), is replaced by an idealized one as depicted in figure 5(b). In the idealized slug flow model the cross-sectional areas in both phases are equal

$$F_L = F_G = F$$

and therefore

$$\mathrm{d}F_L = \mathrm{d}F_G = \mathrm{d}F = 0\,. \qquad [16]$$

Inserting [16] into [5] results in

$$a_{E,L}^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho_L} = a_L^2 \qquad [17]$$

$$a_{E,G}^2 = \frac{\mathrm{d}p}{\mathrm{d}\rho_G} = a_G^2\,. \qquad [18]$$

In contrast to the stratified flow model for which a composite sonic velocity does not exist a compound sonic velocity for an idealized slug flow system is given by

$$a_S = \frac{L}{t} = \frac{L}{t_L + t_G} = \frac{L}{\dfrac{L_L}{a_{E,L}} + \dfrac{L_G}{a_{E,G}}}$$

where subscript S indicates a slug. With the void fraction $\alpha = L_G/L$ the preceding expression becomes

$$a_S = \frac{1}{\dfrac{1-\alpha}{a_{E,L}} + \dfrac{\alpha}{a_{E,G}}}\,. \qquad [19]$$

Substitution of [17] and [18] into [19] yields

$$a_S = \frac{a_L a_G}{(1-\alpha)a_G + \alpha a_L}\,. \qquad [20]$$

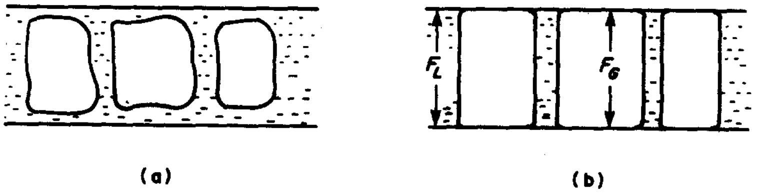

Figure 5. Gas and liquid elements in (a) a slug flow, and (b) an idealized slug flow model.

The sonic velocity in this system, by virtue of the idealized phase distribution, is only a function of the void fraction and the sonic velocities of the pure phases.

Homogeneous flow model

With the assumption of a homogeneous phase distribution according to figure 6 from

$$\alpha = \frac{F_G}{F}$$

follows

$$\mathrm{d}F_G = F\,\mathrm{d}\alpha\,.$$

Together with [7] the above expression converts to

$$\mathrm{d}F_L = -\mathrm{d}F_G = -F\,\mathrm{d}\alpha\,. \qquad [21]$$

Under the assumption that no phase change occurs ($\mathrm{d}x = 0$) and with the defining relation between void fraction and quality x

$$\alpha = \frac{x\rho_L}{x\rho_L + (1-x)\rho_G},$$

the following expression for the variation of the void fraction is obtained

$$\mathrm{d}\alpha = \alpha(1-\alpha)\left(\frac{\mathrm{d}\rho_L}{\rho_L} - \frac{\mathrm{d}\rho_G}{\rho_G}\right). \qquad [22]$$

Now we make use of the previously derived expression for the effective sonic velocity in a fluid with an elastic boundary. Equation [5] applied to the liquid phase of a homogeneous two-phase flow system leads to

$$a_{E,L}^2 = \frac{1}{\dfrac{\rho_L}{F_L}\cdot\dfrac{\mathrm{d}F_L}{\mathrm{d}p} + \dfrac{\mathrm{d}\rho_L}{\mathrm{d}p}}\,.$$

Combining this equation with [1], [21] and [22] results in

$$a_{E,L}^2 = \frac{1}{\dfrac{1-\alpha}{a_L^2} + \dfrac{\alpha\rho_L}{\rho_G a_G^2}}\,. \qquad [23]$$

In an analogous procedure the effective sonic velocity for the gaseous phase of a homogeneous

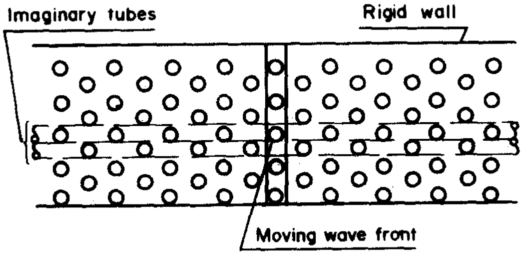

Figure 6. Propagation of an infinitesimal pressure pulse in a homogeneous two-phase system.

two-phase system is derived to be

$$a_{E,G}^2 = \frac{1}{\dfrac{\alpha}{a_G^2} + \dfrac{(1-\alpha)\rho_G}{\rho_L a_L^2}}. \qquad [24]$$

As depicted in figure 6 the system can be divided into imaginary parallel axial tubes. In each tube the wave front serially passes zones of the liquid phase with a velocity $a_{E,L}$ and of the gas phase with the velocity $a_{E,G}$. Therefore the propagation of a pressure pulse in the imaginary tubes corresponds to that in slug flow, so that we can use relation [19] for the evaluation of the composite sonic velocity in homogeneous two-phase flow. Combining [23] and [24] with [19] leads to

$$a_H = \frac{1}{(1-\alpha)\sqrt{\left(\dfrac{1-\alpha}{a_L^2} + \dfrac{\alpha\rho_L}{\rho_G a_G^2}\right)} + \alpha\sqrt{\left(\dfrac{\alpha}{a_G^2} + \dfrac{(1-\alpha)\rho_G}{\rho_L a_L^2}\right)}}. \qquad [25]$$

COMPARISON OF EXPERIMENTAL DATA WITH ANALYTICAL PREDICTIONS

Stratified flow model

Theoretical predictions obtained with [12] and [15] are depicted in figure 7 and compared with experimental data measured by Henry *et al.* (1971). Figure 7(a) gives the comparison for the one-component-system water–vapor and figure 7(b) for the two-component-system water–air. In both cases the predictions made with [12] match the measurements very well, while the curves generated with [15] describing the much lower effective sonic velocity in the liquid phase seem to lack experimental support.

Henry *et al.* (1971) used piezoelectric transducers in their experiments to record the pressure pulse passing two separate locations. Sonic velocity was determined by the distance of the transducers and the time elapsed between the recorded signals. Different experiments with the transducers first being located on the bottom of the horizontal test section under liquid and then on the top in the gas phase led to the same result. If we consider, that in every experiment the first arriving pressure signals were taken for the determination of the effective sonic velocity and that these signals were due to the pressure pulse which passed through the gas phase, the correlations of the experimental and theoretical data shown in figure 7 can be readily explained corresponding to the statements of von Böckh (1975).

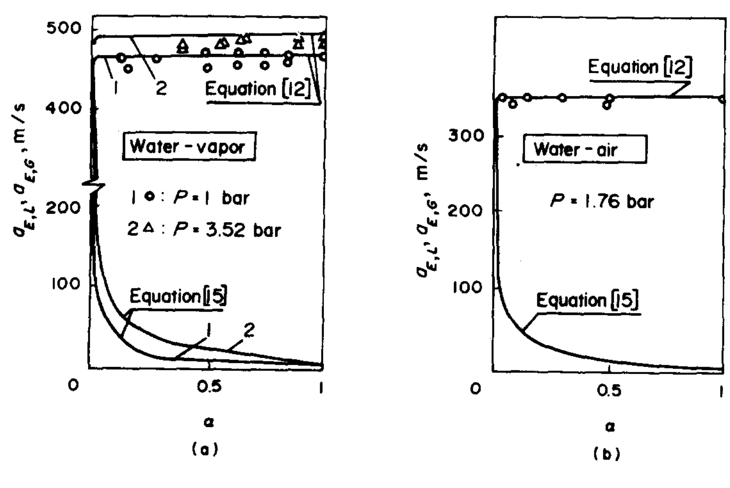

Figure 7. Comparison of experimental data by Henry *et al.* (1971) with analytical predictions.

Slug flow

The idealized slug flow model by Henry *et al.* (1971) and our own one yield identical relations.

Good agreement between theoretical predictions and experimental values by Henry *et al.* (1971) is evident in figure 8.

Homogeneous two-phase flow

Compared to the scarcity of data in other two-phase flow regimes many experimental values are available for homogeneous two-phase flow systems.

The theoretical predictions for homogeneous two-phase flow performed with [25] are in very good agreement with the well known experimental data by Henry *et al.* (1971), England *et al.* (1966) and Karplus (1961) as shown in figures 9–11. Remarkable is the consistency of the measured values with the theory over the entire range of the phase distribution under consideration.

Semenov & Kosterin (1964) apparently are the only experimentators who measured sonic

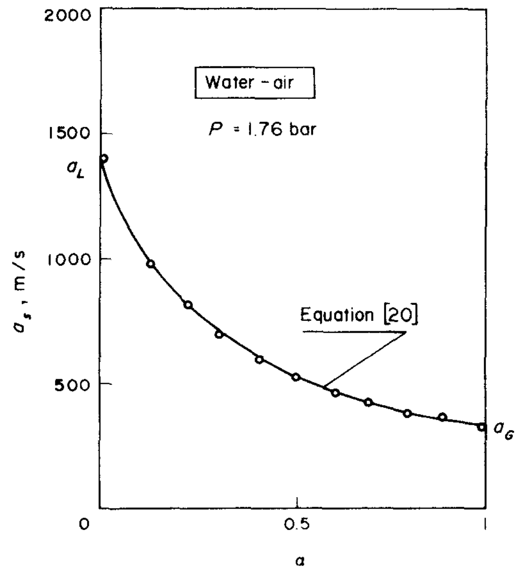

Figure 8. Comparison of analytical with experimental results by Henry *et al.* (1971).

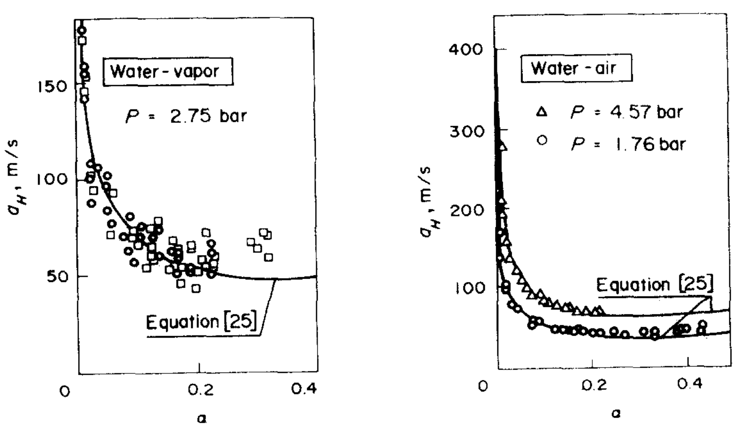

Figure 9. Comparison of experimental data by Henry *et al.* (1971) with analytical predictions.

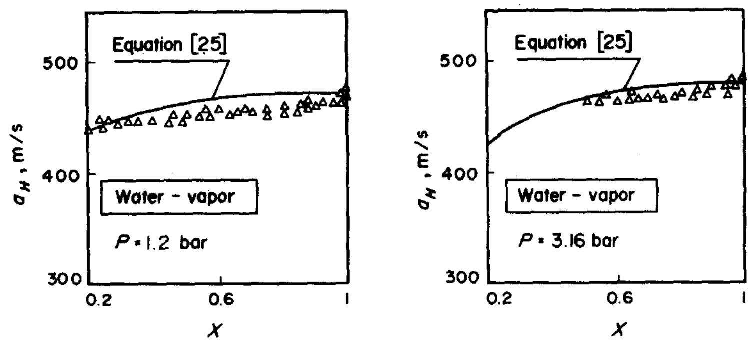

Figure 10. Comparison of experimental data by England *et al.* (1966) with analytical predictions.

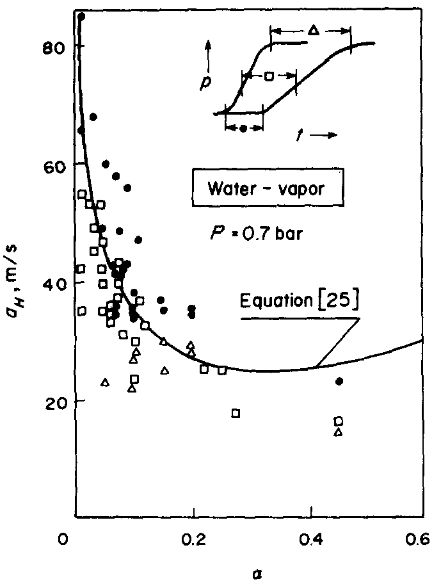

Figure 11.

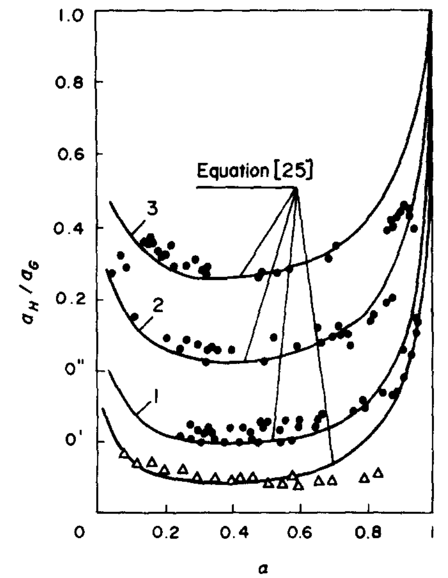

Figure 12.

Figure 11. Comparison of experimental data by Karplus (1961) with analytical predictions.

Figure 12. Comparison of experimental data by Semenov & Kosterin (1964) with analytical predictions. Δ, Water–air system, $p = 1.25$ bar; 1, water–vapor system, $p = 10$ bar; 2, water–vapor system, $p = 15$ bar, coordinate at 0′; 3, water–vapor system, $p = 20$ bar, coordinate at 0″.

velocities in homogeneous two-phase systems over the entire range of void fraction. The predictions with the new model match the experimental values very well in the range $0 < \alpha < 0.6$, although, only marginally well for values of $\alpha > 0.6$. This is true both for one- and two-component homogeneous two-phase flow (figure 12).

CONCLUSIONS

The paper presents a simple model for predicting the sonic velocity in two-phase systems. The model is based upon the well known theory for the computation of the propagation velocity of a pressure disturbance in a single phase fluid bounded by an elastic wall. It is assumed that no phase change occurs during the propagation of a pressure disturbance. The model has been applied to the prediction of sonic velocities in one-dimensional stratified, slug and homogeneous two-phase flow systems. By comparison of the theoretical results with experimental data available in the literature the model has been shown to be remarkably successful in predicting the sonic velocity in most of the flow regimes of two-phase gas–liquid flows over the entire range of void and mass fraction. Currently investigations are in progress to extend the model to the determination of critical mass flow rates in two-phase flow systems.

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